EE 508
Homework 2
Fall 2024
Due Friday Sept 20

Problem 1 Obtain all approximating functions with real coefficients that agree in the magnitude squared sense with one with poles at -1+j1, -1-j1, -4+j8, -4-j8 and with zeros at 2, 2+j6 and 2-j6

Problem 2 Which of those approximating functions in Problem 1 are minimum phase?

Problem 3 Some magnitude-squared functions are given below. If a function T(s) exists that agrees with these functions in the magnitude-squared sense, determine it.

a) $$H(\omega^2)=\frac{1+2\omega^2+6\omega^4}{\omega^8+2\omega^6+3\omega^4+2\omega^2+16}$$

b) $$H(\omega^2)=\frac{1+4\omega^4}{\omega^6+2\omega^4+2\omega^2+1}$$

c) $$H(\omega^2)=\frac{1+\omega+2\omega^2}{\omega^4+2\omega^2+1}$$

d) $H(\omega^2)$ has poles at -1+j1, -1-j1, 1+j1, 1-j1, and a double pole at 2+j0 and zeros at -2+j6, 2-j6, 2+j6, -2-j6 and a double zero at -4+j0

e) $H(\omega^2)$ has poles at -1+j1, -1-j1, 1+j1, 1-j1, - 2+j0 and zeros at -2+j6, 2-j6, 2+j6, -2-j6 and a double zero at -4+j0

Problem 4 Prove or disprove the following theorem:

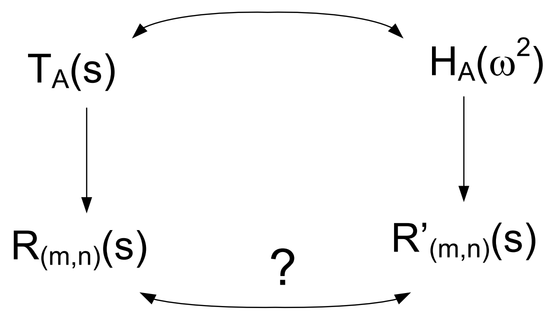
Theorem: If $H_A(\omega^2)$ is the magnitude squared approximation of the all-pole transfer function T(s), then the coefficients of $HA(\omega^2)$ are all of the same sign..

Problem 5 Obtain the 2,3 order Pade approximation $T_{P23}(s)$ of the rational fraction

$$T(s)=\frac{1}{s^5+3.2s^4+5.3s^3+5.3s^2+3.2s+1}$$

and comment on how good the Pade approximation is by comparing the magnitude response of T(s) and $T_{P32}(s)$.

Problem 6 If $R_{(m,n)}(s)$ is the Pade approximation of $T_A(s)$ and $R'_{(m,n)}(s)$ is the Pade approximation of $H_A(\omega^2)$ where $H_A(\omega^2)$ is the magnitude squared function corresponding to transfer function $T_A(s)$, are $R_{(m,n)}(s)$ and $R'_{(m,n)}(s)$ equal? Either prove this relationship or show why it is not true.



Problem 7 $$T(s)=\frac{Ks\frac{\omega_0}{Q}}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$ is the basic second-order bandpass transfer function. Determine the two 3-dB band edges and prove that

a) The magnitude of the gain is a maximum at $\omega=\omega_0$
b) The maximum gain magnitude is K
c) The 3dB bandwidth is ω_0/Q
d) The geometric mean of the two 3-dB band edges is ω_0

Problem 8

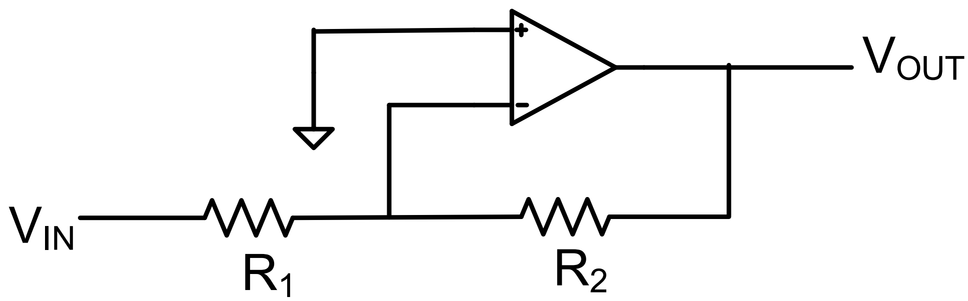
a) What is the minimum value of Q of a pole pair that will result in complex conjugate pole pairs?
b) Derive an expression for the angle of a complex pole relative to the imaginary axis in the s-plane in terms of the parameters ω_0 and Q

Problem 9

a) Using collocation, determine the 6-th order magnitude-squared approximating function with a fourth-order numerator polynomial that has collocation points (0,0.00625),(1,0.00717),(2.5,0.748),(3.5,2.838),(4.5,0.860),(8,0.169)
If it exists, determine the corresponding transfer function $T_A(s)$.
b) Repeat part a) but assume the third point is changed to (2.5,1.0) Comment on the sensitivity to the individual component values

Problem 10

Consider the basic inverting amplifier where the op amp is modeled with a gain function $A(s)=\frac{GB}{s}$. Define $K_0=1+\frac{R_2}{R_1}$.



a) Derive an expression for the frequency dependent gain in terms of the model parameters GB and K_0.
b) Derive an expression for the closed-loop gain bandwidth product in terms of GB and K_0.

Problem 11. Consider the two designs of a finite gain amplifier shown below.
a) Determine resistor values so that the gain of the two designs is 36
b) If the op amps are modeled with a gain $A(s)=\dfrac{GB}{s}$ and GB=1MHz, derive an expression for the 3dB bandwidth of the two designs and comment on the relative performance.